p379 # 19, 20, 33–41, 48, 49, 53, 56, 58, 63

(19) $\dfrac{\sin(-x)}{\cos(-x)} = \dfrac{-\sin x}{+\cos x} = -\tan x$ (E)

(20) $\dfrac{\sin(\pi/2 - x)}{\cos(\pi/2 - x)} = \dfrac{\cos x}{\sin x} = \cot x$ (C)

(33) $\dfrac{1-\sin^2 x}{\csc^2 x - 1} = \dfrac{\cos^2 x}{\cot^2 x} = \dfrac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} = \sin^2 x$

(34) $\dfrac{1}{\tan^2 x + 1} = \dfrac{1}{\sec^2 x} = \cos^2 x$

(35) $\sec \alpha \cdot \dfrac{\sin \alpha}{\tan \alpha} \; \frac{\sin}{\cos} = \dfrac{1}{\cos \alpha} \cdot \cos \alpha = 1$

(36) $\dfrac{\tan^2 \theta}{\sec^2 \theta} = \cos^2 \theta \cdot \dfrac{\sin^2 \theta}{\cos^2 \theta} = \sin^2 \theta$

(37) $\cos\left(\frac{\pi}{2} - x\right)\sec x = \sin x \dfrac{1}{\cos x} = \tan x$

(38) $\cot\left(\frac{\pi}{2} - x\right)\cos x = \tan x \cos x = \dfrac{\sin x}{\cos x} \cdot \cos x = \sin x$

(39) $\dfrac{\cos^2 y}{1-\sin y} = \dfrac{1-\sin^2 y}{1-\sin y} = \dfrac{(1-\sin y)(1+\sin y)}{1-\sin y} = 1+\sin y$

(40) $\cos t(1+\tan^2 t) = \cos t(\sec^2 x) = \dfrac{\cos t}{\cos^2 t} = \dfrac{1}{\cos t} = \sec t$

(41) $\sin\beta \tan\beta + \cos\beta = \sin\beta \dfrac{\sin\beta}{\cos\beta} + \cos\beta = \dfrac{\sin^2\beta}{\cos\beta} + \dfrac{\cos^2\beta}{\cos\beta}$

$= \dfrac{1}{\cos\beta} = \sec\beta$

(48) $\cos^2 x + \cos^2 x \tan^2 x = \cos^2 x(1+\tan^2 x)$

$= \cos^2 x(\sec^2 x)$

$= \cos^2 x\left(\dfrac{1}{\cos^2 x}\right) = 1$

(49) $\dfrac{\sec^2 x - 1}{\sec x - 1} = \dfrac{(\sec x - 1)(\sec x + 1)}{\sec x - 1} = \sec x + 1$

(53) $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$= (\sin^2 x - \cos^2 x)1 = \sin^2 x - \cos^2 x$

(56) $\sec^3 x - \sec^2 x - \sec x + 1$

$= \sec^3 x - \sec^2 x - (\sec x - 1)$

$= \sec^2(\sec x - 1) - (\sec x - 1)$

$= (\sec x - 1)(\sec^2 x - 1) = (\sec x - 1)\tan^2 x$

(58) $(\cot x + \csc x)(\cot x - \csc x)$

$= \cot^2 x - \csc^2 x = -1$

(63) $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x} = \dfrac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)(\cos x)}$

$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x} = \dfrac{2 + 2\sin x}{(1+\sin x)\cos x}$

$= \dfrac{2(1+\sin x)}{(1+\sin x)\cos x} = \dfrac{2}{\cos x} = 2\sec x$